Archimedes and the Golden Crown

Archimedes was a Greek scientist and mathematician who lived in Syracuse, Sicily in the 3rd Century BC. He was born in 287BC and died in 212BC. He is considered one of the greatest mathematicians of all time, and is credited with many inventions that we still use today.

He is famous for

- determining that π was somewhere between $3\frac{10}{71}$ and $3\frac{10}{70}$ (which is $3\frac{1}{7}$; our familiar 22/7 is, of course, $3\frac{1}{7}$)
- inventing the Archimedes screw, a type of water pump;
- inventing the compound-pulley system;
- determining the mathematical laws that govern levers and pulleys;
- formulating the Archimedes Principle (which describes how much force an object which is wholly or partially immersed in a fluid is subject to).

He also made many other mathematical discoveries which were not improved upon for nearly 2000 years.

A famous story tells of the King of Syracuse, Hieron, who asked a goldsmith to make a golden crown for him. The king later suspected that the crown was not made of pure gold, and that the goldsmith had stolen some of the gold and replaced it with silver. King Hieron asked Archimedes to work out if the crown was made of pure gold, but he wasn't allowed to melt it down, since the crown had already been dedicated to the gods. While stepping into the bath one day, he realized how to test the crown and ran out into the street shouting "eureka", which means "I found it".

So how did Archimedes work out how to test the crown? The activities and pracs in the next few lessons will reveal all.
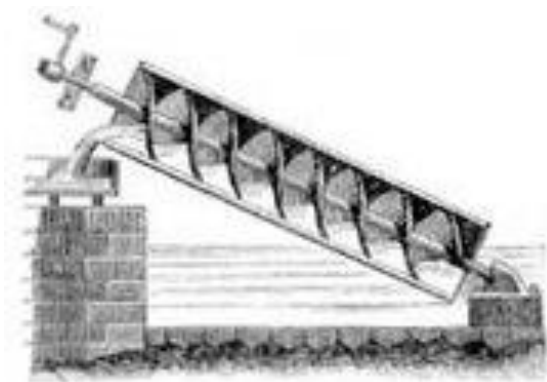
A cut-away picture showing an Archimedes Screw
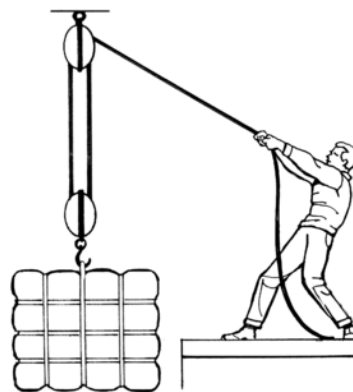
A compound-pulley system.

Density   Name: _____

Introduction: The density of a substance is a measure of its mass per unit of volume. Its most common unit is grams per cubic centimetre (g/cm^3) or grams per millilitre (g/mL). Note: 1 cm^3 = 1 mL.

The formula for density is: density = mass/volume

Aim: To measure the density of a variety of materials by measuring the materials' mass and volume.

Equipment: Spring balance or scales, ruler, measuring cylinder, beaker, cubes of lead, brass, wood, aluminium and zinc, glass block.

Part A. Density of Solids

Substance	Mass (g)	Volume (cm^3) (length x width x height)	Measured Density (g/cm^3)	Actual Density (g/cm^3)	% error (of your measurement to actual value) $\frac{(actual\ value - calculated\ value)}{actual\ value}$ x 100%
Lead				11.34	
Brass				Approx 8.4 -8.8	-------------
Wood				varies a lot	-------------
Aluminium				2.7	
Zinc				7.14	
Glass				varies a lot	-------------

Part B. Density of Liquids

On a set of scales find the mass of a beaker.   Mass of beaker: _____

Using a measuring cylinder, pour exactly 100ml of water into the beaker.

Find the mass of the water and beaker combined.   Mass of beaker and water: _____

Find the mass of just the water.   Mass of water: _____

Density of water = $\frac{mass\ of\ water}{volume\ of\ water}$ = $\frac{}{100mL}$ = g/cm^3

Questions:

1. Which material was the most dense? _____

2. Which material was the least dense? _____

3. How accurate were your results for lead, aluminium and brass compared to the actual values?

_____
_____
_____

4. The gram and the kilogram were originally defined with respect to water: <u>by definition</u>, 1cm^3 of water had a mass of 1 gram. Water's density is therefore 1g/cm^3. How accurate were your results?

_____
_____
_____

5. So, 200mL of pure water should have a mass of _____grams, while 500mL of water should have a mass of _____grams.

6. When scientists set up new labs, they often test their equipment and their experimental techniques with stuff they already know about (to ensure that everything is correct), and only then do they begin trying to make new discoveries. What do your answers in Qs 3 & 4 say about your experimental technique and the equipment you used?

_____
_____
_____
_____

7. The formula density = mass/volume is often written as $\rho = m/V$ The "ρ" symbol is the Greek character "rho" and is pronounced like "row". ρ = density, m = mass and V = volume. If $\rho = m/V$ what is m in terms of V and ρ.

8. Calculate the mass of 1000cm^3 of lead.

9. Calculate the mass of a 10cm x 10cm x 5cm block of aluminium.

10. The density of gold is 19.3 g/cm^3. How much would a block 10cm x 5cm x 5cm weigh?

11. What would the volume of 1kg of gold be?

12. What would the side length of a 1kg gold cube be? Draw a life-size 3D representation of the cube below.

Volume of Solids.   Name: _____

Every object takes up space. The space it takes up is called its volume. The volume of a rectangular prism is given by the formula Volume = length × width × height. The volume of a cylindrical prism is given by the formula $V = \pi r^2 h$.

How can we find the volume of an irregular solid (like a rock or a clay model)? When objects are lowered into water, they "displace" a certain amount of water. If a glass of water is full and you put your fingers into it, some of the water will spill out.
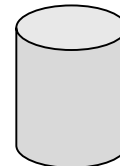
How does the volume of the water that spills out compare to the volume of the object?

Aim: To determine the amount of water displaced by objects of a known volume.
Equipment: metal blocks, displacement beakers, scales, measuring cylinders.
Method:
Fill a displacement beaker to the brim and then let it rest near a sink. Let the water flow out of the side tube. Keep the beaker still.
Carefully lower the object (or objects) into the water. You will need to tie a thin cotton thread around the object and lower it in. The water will overflow. Collect the water that overflows with a measuring cylinder.
To help achieve accuracy, you will have to repeat each test three times and then take an average.
Fill in the table.

Object	Length	Width	Height	Volume (V =lwh)	Volume of water displaced.			
					Trial 1	Trial 2	Trial 3	Average
1 cube								
2 cubes								

Object	Height	Radius	Volume ($V = \pi r^2 h$)	Volume of water displaced.			
				Trial 1	Trial 2	Trial 3	Average
Cylindrical prism							

Questions:
1. How does the volume of water displaced compare with the volume you calculated? What errors could have occurred?

    _____
    _____
    _____
    _____

2. Why do scientists usually repeat their experiments a number of times?

    _____
    _____
    _____

3. How can you measure the volume of an irregular solid like a rock?

    _____
    _____
    _____

Density of Irregular Solids.   **Name:** _____

Find the density of the following substances by first finding their mass and their volumes.

Substance	Mass (grams)	Volume (mL or cm^3)			Density ($density = \frac{mass}{volume}$)
		Trial 1	Trial 2	Average	
Glass					
Plastic					
Rock					
Clay					
Rubber					

Questions:

1. How does the density of non-metallic substances compare to the density of most metals?
   _____
   _____
   _____
   _____

2. Collect data from your class mates about the density of 2 substances.

	Density (g/cm^3)							
Substance	Your results	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7	Average

3. Comment on the precision of the groups' results. What does this tell you about the class's experimental technique? How confident are you that the values you calculated are the actual densities of the substances?
   _____
   _____
   _____
   _____
   _____
   _____

4. Describe how Archimedes could have determined if the crown was pure gold or not.
   _____
   _____
   _____
   _____
   _____
   _____

**Buoyancy**   Name: _____

When an object is immersed in water, the water doesn't simply get displaced; the water provides an upwards force on the object. This upwards force is called **buoyancy**. (The normal downwards force is called **gravity**.) All objects therefore <u>seem</u> lighter when immersed in water. **How much buoyancy does the water provide?** Why do some things float while others sink?

Aim: To determine the buoyancy on various objects when immersed into water.

Equipment: spring balances, cotton thread, overflow beaker, measuring cylinder, brass cube, zinc cube, brass cylindrical prism, and wooden cube.

Method: Find the weight (in grams) of each object using a spring balance. Lower the metal cube into the water, ensuring that it does not touch the bottom. Record the new (apparent) weight (in grams).

At the same time, measure the amount of water that has been displaced by the object. Fill in the table. Repeat for the cylindrical prism and the wood.

Overflow can
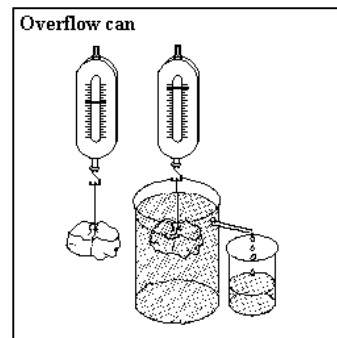
Object	weight in air (grams)	Apparent weight in water (grams)	Difference in weight (grams)	Force of Buoyancy (grams) (same as previous column!)	Volume of water displaced. (mL) (this volume should be the same as the volumes you found in the "Volume of Solids" prac.)	Weight of water displaced. (grams) (same as previous column since water's density is 1g/mL)
Brass cube						
Zinc cube						
Brass cylindrical prism						
Wood						

Questions:

1. How does the force of buoyancy compare with the weight of water displaced?

_____
_____
_____

2. The volume of the object is equal to the volume of the water displaced. How does the volume of the object compare to the force of buoyancy?

_____
_____
_____

3. Explain **two** ways of determining the volume of an irregular object.

(a)

_____
_____
_____

(b)

_____
_____
_____

For the following **three** questions, cross out the wrong option and circle the correct option.

4. An object wholly or partly immersed in water will experience an upwards/downwards force equal to/greater than/less than the weight of the water displaced. (This principle is actually called **"Archimedes' Principle"**.)

5. If the object floats, the buoyancy force is greater/less than its weight (the weight is the force of gravity). Its density is less/greater than water's density (which is 1g/cm³).

6. If the object sinks, the buoyancy force is greater/less than its weight. Its density is less than / greater than water's density (1 g/cm³).

7. Comment on the buoyancy of the brass cube and the zinc cube.

_____
_____
_____

8. A golf ball weighing 46 grams sinks while a tennis ball weighing 57 grams floats. Explain why.

_____
_____
_____

$density = \frac{mass}{volume}$ $\qquad \rho = \frac{m}{V}$ (ρ = density in g/cm³ m = mass in grams (g) V = volume in cm³)

9. A solid piece of plastic has a mass of 150 grams. It is 5cm long, 2 cm wide and 10 cm high. Calculate its **volume** and its **density,** and then state whether it would **float or sink**.

10. Most people can float when they fill their lungs with air but sink when they completely exhale. Why?

_____
_____
_____

11. A piece of aluminium has a mass of 170 grams. If aluminium's density is 2.7 grams/cubic centimetre, calculate
(a) its volume

(b) the volume of water that it displaces when submerged

(c) the weight of the water displaced (in grams)

(d) the force of buoyancy on it (in grams) when it is submerged

(e) the apparent weight (in grams) when submerged

12. A piece of an unknown metal weighs 500 grams, but when it is submerged, it appears to weigh 400g.
Calculate (a) the force of buoyancy;

(b) the weight of the water displaced;

(c) the volume of the piece of metal;

(d) the density of the metal